The Employment-Productivity Relationship with Employment Criteria*

Sumati Srinivas
Department of Economics
Central Michigan University
Mt. Pleasant, MI 48858

Michael Sattinger
Department of Economics
University at Albany
Albany, NY 12222

May 7, 2003

Abstract

This paper analyzes labor market responses to productivity shocks when firms set employment criteria on the basis of the likelihood of hiring high or low productivity workers. In response to a positive productivity shock, firms do not raise the criterion as much as the shock, increasing the proportion of low productivity workers among the employed. The observed average productivity may respond negligibly even if employment changes substantially. Interest rate fluctuations can yield an opposite relation between productivity and employment, explaining the weak empirical relationship.

1 Introduction

This paper proposes an alternative view of the labor market to explain weak and contradictory relations between employment and productivity over the business cycle. Instead of choosing an amount of labor to employ at a particular real wage, firms set an employment criterion to determine which applicants to hire. In contrast to the implications of the standard model with

*The authors are indebted to Betty Daniel, Michael Jerison and Thad Mirer for helpful comments. Remaining errors are the responsibility of the authors.

homogeneous labor, the employment criterion approach yields the following major results.

- In response to a productivity shock, observed average productivity varies less than proportionately with the initial productivity shock, so that large employment fluctuations can be associated with small observed productivity changes.

- Average productivity can be positively or negatively related to employment, depending on whether the fluctuation in the economy is from a productivity shock or a monetary disturbance (affecting the interest rate).

In the Real Business Cycle approach initially developed by Kydland and Prescott (1982), productivity shocks are the source of aggregate fluctuations in employment and economic activity (see the review by Hartley, Hoover, and Salyer, 1998). One question considered in the literature is whether adjustments to productivity shocks would generate employment fluctuations of the magnitudes observed empirically. Hansen (1985) shows that large fluctuations in employment can be generated by the model if labor is assumed to be indivisible (see also Hansen and Wright, 1992). A related question is why the posited relation between productivity shocks and aggregate fluctuations is not reflected in empirical estimates of the correlation between productivity and employment (measured in hours worked). Low correlation between productivity and employment has been explained within the literature by introducing other sources of shocks. Benhabib, Rogerson and Wright (1991) introduce household production and home production shocks, and Christiano and Eichenbaum (1992) introduce government spending shocks (see also Hansen and Wright, 1992).

This paper proposes an alternative resolution of these issues. Instead of choosing an amount of labor, firms choose an employment criterion. Workers are heterogeneous and have either high productivity or low productivity. Firms observe worker productivity imperfectly and set an employment criterion for the observed worker productivity to determine which workers to employ. In response to a productivity shock that increases worker productivities proportionately, firms raise the employment criterion less than proportionately to the shock and may even lower it. Employment expands along with a decrease in the proportion of high productivity workers among

the employed, which counters the increase in average productivity from the productivity shock. The result can be a substantial increase in employment associated with a negligible change in observed average productivity. As a simple, paradigmatic model of the labor market, the analysis developed here abstracts from dynamics and expectations that are important in fully delineated macroeconomic models.

Heterogeneous labor has been introduced previously in the study of business cycle fluctuations (Kydland, 1984; King, Plosser and Rebelo, 1988; Hansen and Sargent, 1988; Cho and Rogerson, 1988; and Cho, 1995). Prasad (1996) investigates aggregation bias in measurements of productivity and the real wage from ignoring skill heterogeneity in workers. Employment criteria or hiring standards have been considered in labor economics as an alternative margin of adjustment over the business cycle. Gaston (1972) shows that hiring standards vary depending on labor market conditions. Thurow (1975) develops the concept of an employment criterion in developing a theory of job competition to explain statistical discrimination (see also Schlicht, 1981). Lockwood (1991) analyzes a matching model in which firms observe worker productivity imperfectly and use unemployment duration as an employment criterion. Schlicht (2002) develops a model in which firms simultaneously set wage offers and hiring standards, generating results that differ from efficiency wage models.

Section 2 develops the model, beginning with firm behavior. While the proportion of unemployed workers is exogenous to the individual firm, it is endogenously determined in the market. Incorporating the determination of this proportion into the firm first order condition yields a market condition for equilibrium, permitting comparative statics. Section 3 introduces productivity shocks, which change worker productivities proportionately without changing firm abilities to distinguish between high and low productivity workers. It is shown that the employment criterion increases less than proportionately to a productivity shock, and that average productivity increases by a smaller proportion than the productivity shock. Section 4 considers fluctuations in the interest or discount rate as a source of shock to the labor market, presumably generated by monetary disturbances. Since the benefits of employing workers are distributed over time, a change in the discount rate leads firms to alter their employment criteria. It is shown that an interest rate increase reduces employment while raising average productivity, since the employment criterion becomes stricter. This is the opposite relationship from a productivity shock. Section 5 discusses the conclusions.

2 Model

2.1 Firm Determination of Employment Criterion

The labor market is characterized as follows. Identical firms interview workers and decide on the basis of a test or interview whether a worker is likely to have high or low productivity. Firms decide to hire a worker if the expected profit from the worker is positive (or non-zero). It will be shown that firms set an employment criterion and hire any worker with a test score greater than or equal to that employment criterion. Workers have a common quit rate but different hiring rates depending on whether they have high or low productivity. The two types of workers then have unequal unemployment rates, which determine endogenously the proportion of high productivity workers among the unemployed. This section shows how the employment criterion is determined and examines its existence and uniqueness.

First consider the problem facing the firm. Suppose workers either have high productivity, p_1, or low productivity, p_2, with $p_1 > p_2$. Suppose the firm observes the productivity imperfectly as in the statistical discrimination literature:

$$y_i = p_i + \epsilon_i \tag{1}$$

where p_i is the productivity of worker i, y_i is the observed productivity for worker i, and ϵ_i is an independently and identically distributed random error term. Suppose ϵ_i is distributed normally with mean 0 and variance σ^2, and let $f(\epsilon)$ and $F(\epsilon)$ be the probability density function and cumulative distribution function, respectively.

Let w be the wage rate, taken as given in this section (Section 4.3 will consider the determination of the wage rate). The wage is the same for all workers. Let q be the quit rate, the same for all workers, and let r be the discount rate, the same for all firms. Suppose firms incur a cost of c for all workers hired. By integration, the present discounted value from hiring a worker with productivity p_i is

$$\pi_i = \frac{p_i - w}{q + r} - c, i = 1, 2 \tag{2}$$

It is assumed that $\pi_1 > 0 > \pi_2$. If instead $\pi_1 > \pi_2 > 0$, the firm would hire all workers, and if $0 > \pi_1 > \pi_2$, the firm would hire no workers.

Now consider the likelihood that a worker with observed productivity (or score) y_i has high productivity, p_1. If the worker actually has productivity

p_1, then $\epsilon_i = y_i - p_1$, while if the worker actually has productivity p_2, then $\epsilon_i = y_i - p_2$. Let μ be the proportion of high productivity workers among the unemployed. (This will be determined endogenously by firm hiring decisions, but a single firm's decision will not affect μ so the firm will take μ as given.) Applying Bayes Rule, the likelihood that a worker with score y_i is high productivity is

$$\frac{\mu f(y_i - p_1)}{\mu f(y_i - p_1) + (1-\mu) f(y_i - p_2)} \tag{3}$$

The probability that the worker is low productivity is one minus the amount in 3. The expected added profit from hiring worker i is then

$$E(\pi_i) = \frac{\mu f(y_i - p_1)\pi_1 + (1-\mu) f(y_i - p_2)\pi_2}{\mu f(y_i - p_1) + (1-\mu) f(y_i - p_2)} \tag{4}$$

The firm should hire the worker whenever $E(\pi_i) \geq 0$.[1]

The following argument shows that there will be a threshold value of y_i, the employment criterion y_0, such that it will be profitable for the firm to hire all workers with $y_i \geq y_0$. A consequence of the normality assumption is that $d \log f(\epsilon)/d\epsilon = -2\epsilon/2\sigma^2$, a decreasing function of ϵ. Then

$$\frac{d \log(f(y_i - p_1)/f(y_i - p_2))}{dy_i} = \frac{f'(y_i - p_1)}{f(y_i - p_1)} - \frac{f'(y_i - p_2)}{f(y_i - p_2)} > 0 \tag{5}$$

so that $f(y_i - p_1)/f(y_i - p_2)$ is an increasing function of y_i. Let y_0 be the value of y_i such that the numerator in 4 is zero:

$$\mu f(y_0 - p_1)\pi_1 + (1-\mu) f(y_0 - p_2)\pi_2 = 0 \tag{6}$$

The value y_0 can be shown to exist.[2] Then if $y_i > y_0$,

$$\frac{f(y_i - p_1)}{f(y_i - p_2)} > \frac{f(y_0 - p_1)}{f(y_0 - p_1)} \tag{7}$$

[1] A worker with $y_i = y_0$ will yield zero expected profit to the firm. Such workers will have measure zero. As a convention, it will be assumed that firms hire them too.

[2] Existence can be demonstrated as follows using features of the normal distribution. At low values of y_0, $f(y_0 - mp_1)/f(y_0 - mp_2)$ will approach zero, so that the left side will be negative (since $\pi_2 < 0$). As y_0 increases indefinitely, the ratio $f(y_0 - mp_1)/f(y_0 - mp_2)$ will increase indefinitely and the left hand side will be positive since $\pi_1 > 0$. By continuity, there will be a value y_0 such that the left hand side of the equation is zero.

so that $E(\pi_i) > 0$. Therefore the firm will hire all workers with $y_i \geq y_0$. The firm's strategy concerning which workers to hire is analogous to the reservation wage property in search theory.

Because of the foregoing result, it is possible to reformulate the firm's problem as follows. Let β be the number of interviews per period for the firm (assumed to be exogenous to the firm) and let C_I be the cost per interview. Then the firm's expected profit per period is:

$$E(\pi) = \beta\left(\mu[1 - F(y_0 - p_1)]\pi_1 + (1 - \mu)[1 - F(y_0 - p_2)]\pi_2 - C_I\right) \qquad (8)$$

The firm maximizes $E(\pi)$ with respect to y_0, yielding the first order condition in 6. The second order condition, after applying the first order condition and rearranging, is given by 5 and is satisfied because of the normality assumption.

The firm's problem in choosing the employment criterion y_0 can be understood as follows. Any unemployed worker appearing for an interview could be either a high productivity or a low productivity worker. The firm is willing to risk hiring a low productivity worker (and losing money on that worker) if there is a sufficient chance of getting a high productivity worker and making money. If $y_i = y_0$, the addition to profits from the chance of hiring the high productivity worker just balances the loss from the risk of hiring a low productivity worker, and the expected gain in profit from hiring the worker is zero. At any higher value of y_i, the expected profit from hiring a high productivity worker outweighs the risk of loss, and the expected added profit is positive.

2.2 Proportions of the Employed and Unemployed That Are High Productivity

Firms, in choosing the employment criterion $y_{0,}$ take μ, π_1, and π_2 as given. However, μ, the proportion of the unemployed that are high productivity, depends on the hiring decisions of firms. This section examines how μ is determined.

Suppose workers receive interviews at the rate of θ per period. A worker with productivity p_i will have $y_i < y_0$ in a proportion $F(y_0 - p_i)$ of interviews. Then the proportion of interviews that yield job offers is $1 - F(y_0 - p_i)$. The rate at which an unemployed worker with productivity p_i gets a job is therefore $\theta[1 - F(y_0 - p_i)]$. Let u_i be the unemployment rate for workers

with productivity p_i. The long run equilibrium level of unemployment will be achieved when the flow of workers from employment to unemployment, $(1-u_i)q$, equals the flow from unemployment to employment:

$$(1-u_i)q = u_i\theta[1-F(y_0-p_i)], i=1,2 \tag{9}$$

Then

$$u_i = \frac{q}{q+\theta[1-F(y_0-p_i)]}, i=1,2 \tag{10}$$

Let ρ be the proportion of high productivity workers among the population. The proportion of workers unemployed in equilibrium is $\rho u_1 + (1-\rho)u_2$. The proportion of unemployed workers that are high productivity is then

$$\mu = \frac{\rho u_1}{\rho u_1 + (1-\rho)u_2} \tag{11}$$

2.3 Market Determination of Employment Criterion

The previous derivations can now be combined to yield the following theorem on the market determination of the employment criterion.

Theorem 1 *In the Employment Criterion Model, the criterion y_0 exists and satisfies the following condition:*

$$\frac{\rho}{1-\rho}\frac{q+\theta[1-F(y_0-p_2)]}{q+\theta[1-F(y_0-p_1)]}\frac{f(y_0-p_1)}{f(y_0-p_2)} = \frac{-\pi_2}{\pi_1} \tag{12}$$

Proof: Using 11 to substitute for μ in the firm first order condition 6 yields

$$\frac{\rho u_1 f(y_0-p_1)\pi_1 + (1-\rho)u_2 f(y_0-p_2)\pi_2}{\rho u_1 + (1-\rho)u_2} = 0 \tag{13}$$

Multiplying by $\rho u_1 + (1-\rho)u_2$, substituting for u_1 and u_2 from 10 and rearranging yields 12. When this condition is satisfied, firms' first order conditions are satisfied, and μ is consistent with firm choices of y_0. The value of y_0 satisfying 12 is such that the labor market is in equilibrium.

Next, consider existence and uniqueness of the employment criterion satisfying 12. The strategy of the proof can be demonstrated using Figure 1. The upward-sloping curve in the figure shows the left hand side of 12 as a

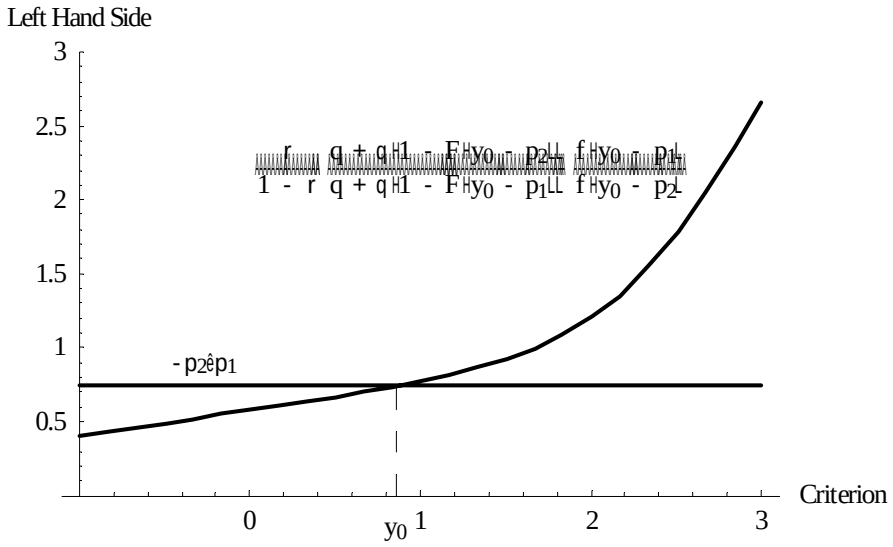

Figure 1: Market Equilibrium Condition

function of y_0.[3] The horizontal line is at the ratio of profits $-\pi_2/\pi_1$, where π_2 is negative. The value of y_0 where the curve reaches $-\pi_2/\pi_1$ is the equilibrium value. If the left hand side of 12 is monotonically increasing, starting below $-\pi_2/\pi_1$ and going above that value, then existence and uniqueness would follow immediately. While $f(y_0 - p_1)/f(y_0 - p_2)$ increases monotonically because of the assumption of normality,

$$\frac{u_1}{u_2} = \frac{q + \theta[1 - F(y_0 - p_2)]}{q + \theta[1 - F(y_0 - p_1)]} \tag{14}$$

does not. It is therefore necessary to investigate the individual components of 12 that depend on y_0.

From the normal distribution,

$$\begin{aligned} \frac{f(y_0 - p_1)}{f(y_0 - p_2)} &= \frac{(1/\sqrt{2\pi}\sigma e^{-(y_0 - p_1)^2/(2\sigma^2)})}{(1/\sqrt{2\pi}\sigma e^{-(y_0 - p_2)^2/(2\sigma^2)})} \\ &= Exp((p_1 - p_2)(2y_0 - p_1 - p_2)/(2\sigma^2)) \end{aligned} \tag{15}$$

[3]The parameters for this figure are $\sigma = 1$, $q = .1$, $w = .6$, $r = .05$, $p_1 = 1$, $p_2 = .5$, $\rho = .5$, $c = .75$, $C_I = .2$ and $\mu = .4$. The equilibrium value of y_0 is .854 and the value of μ is .412.

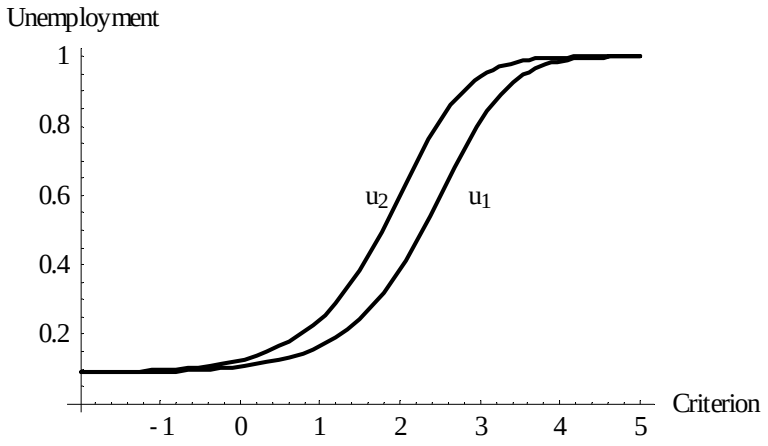

Figure 2: Unemployment Rates

where $Exp(x) = e^x$. Since $p_1 > p_2$, this is an increasing exponential function of y_0, starting at zero for indefinitely small y_0 and increasing indefinitely as y_0 increases indefinitely.

Now consider u_1/u_2. Figure 2 shows the behavior of the unemployment rates individually and Figure 3 shows the ratio u_1/u_2. From the functional forms, this ratio first decreases over an interval and then increases. Since the unemployment rates approach each other at arbitrarily low and high values of y_0, the ratio u_1/u_2 starts at one and ends at one as y_0 increases over its range. Then the left-hand side of 12 starts at zero for sufficiently low y_0 (and is less than $-\pi_2/\pi_1$) and eventually goes above $-\pi_2/\pi_1$ for sufficiently large y_0. By continuity, the left hand side of 12 must equal $-\pi_2/\pi_1$ at some value of y_0, establishing existence. This completes the proof of Theorem 1.

The theorem does not include a statement of uniqueness because of complications. The ratio $f(y_0 - p_1)/f(y_0 - p_2)$ is monotonically increasing but u_1/u_2 is not. However, if the solution for y_0 yields sufficiently low unemployment rates, the slope of u_1/u_2 approaches zero so that the product of the two ratios is monotonically increasing in a lower range for y_0. Also, it can be shown that changes in the ratio $f(y_0 - p_1)/f(y_0 - p_2)$ dominate changes in u_1/u_2, so the left hand side of 12 is a monotonically increasing function of the employment criterion, y_0. The market equilibrium value of the employment criterion would then be unique.

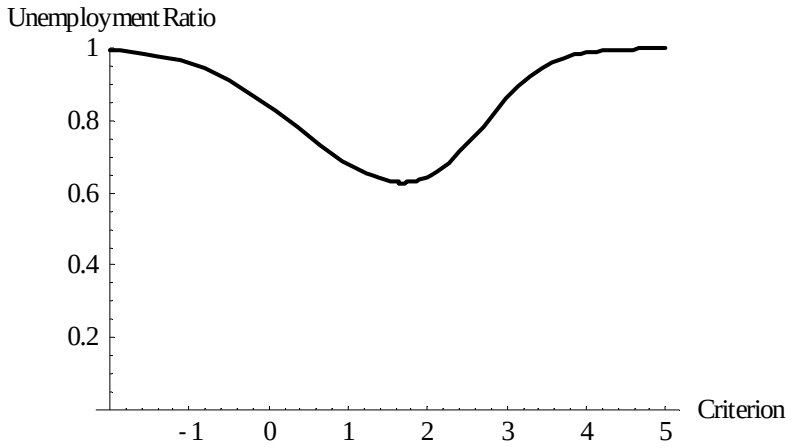

Figure 3: Ratio of Unemployment Rates, u_1/u_2

2.4 Comparative Statics

The condition for the equilibrium employment criterion in 12 yields the following comparative statics results:

Theorem 2 *The employment criterion y_0 will be greater when c, w or θ are greater or when r, ρ, p_1 or p_2 are less.*

Proof. The equilibrium employment criterion occurs when the left-hand side of 12 crosses the horizontal line at the level $-\pi_2/\pi_1$. When the left-hand side of the condition shifts down or $-\pi_2/\pi_1$ goes up, the employment criterion increases. As c or w increase, π_1 decreases and π_2 decreases, so that $(-\pi_2)$, a positive amount, increases. Then the ratio $-\pi_2/\pi_1$ increases, leading to an increase in y_0. Similarly, an increase in the interest rate r raises $-\pi_2/\pi_1$, resulting in an increase in y_0. Declines in either p_1 or p_2 raise $-\pi_2/\pi_1$, leading to a greater value of y_0. The parameters ρ and θ affect the left-hand side of the condition but not $-\pi_2/\pi_1$. If ρ declines, the left-hand side shifts down. If θ increases, the ratio u_1/u_2 declines since $1 - F(y_0 - p_1) > 1 - F(y_0 - p_2)$, shifting the left-hand side of the condition down and increasing y_0. This completes the proof.

In general, parameter changes that raise the profitability of a marginal applicant (with criterion equal to y_0) lead firms to risk hiring more low pro-

ductivity workers (by lowering the criterion y_0) in order to hire more high productivity workers. Comparative static effects of parameter changes are relevant to the analysis of labor market responses to cyclical conditions, which will be examined in Section 4.

3 The Effects of Productivity Shocks

3.1 Assumptions

At this point it is possible to introduce productivity shocks into the model. In the RBC models, positive productivity shocks raise the demand for labor. Then at a constant wage rate, employment increases along with an increase in the average productivity of labor. In the model developed here (with heterogeneous labor, imperfect observation of worker productivity and employment criteria), the net response depends on the adjustment in the employment criterion. It will be shown that the employment criterion adjusts less than the productivity shock, raising the proportion of low productivity workers among the employed and moderating the observed productivity change. The change in average productivity may be negligible in comparison to the productivity shock.

Productivity shocks are assumed to affect all workers' productivities by the same proportion. Then in 1, the productivity of a worker of type i is

$$p_i s, \ i = 1, 2 \tag{16}$$

where s is the productivity shock and p_1 and p_2 are the productivities of the high and low productivity workers when s equals one. The standard deviation of the error term in 1 also changes in proportion to the shock, so that the shock has no effect on the ability of firms to distinguish between high and low productivity workers. This assumption is fully consistent with the type of productivity shock assumed in RBC models. To incorporate this assumption, write the probability density function for the normally distributed error term in 1 as $f(\epsilon; \sigma)$, where σ is the standard deviation. In response to a productivity shock s, the probability density function and cumulative distribution function do not change values if ϵ increases by the same proportion as s. This is achieved if the probability density function and cumulative distribution function are $f(\epsilon; s\sigma)$ and $F(\epsilon; s\sigma)$. Then the assumption concerning

the productivity shocks yields

$$f(s\epsilon; s\sigma) = f(\epsilon; \sigma), F(s\epsilon; s\sigma) = F(\epsilon; \sigma) \tag{17}$$

3.2 Effects of Productivity Shocks on the Employment Criterion

With productivity shocks given by 16, the test scores in 1 will be raised by a positive productivity shock. More of both types of workers will have scores that exceed the existing employment criterion, y_0. However, the employment criterion will also adjust, as described in the following theorem.

Theorem 3 *In response to a positive productivity shock $s > 1$ (starting from $s = 1$), the ratio of the equilibrium employment criterion to the parameter s declines. Unemployment rates of both types of workers decline but if the unemployment rates initially are sufficiently low, the ratio u_1/u_2 increases, and the proportion of high productivity workers among the unemployed, μ, increases.*

Proof. The proof proceeds by considering whether an increase in the employment criterion proportional to s satisfies the market condition 12. Let $\widehat{y_0}$ be the initial value of the employment criterion. By construction, $f(\widehat{y_0}s - p_i s; s\sigma) = f(\widehat{y_0} - p_i; \sigma)$ and $F(\widehat{y_0}s - p_i s; s\sigma) = F(\widehat{y_0} - p_i; \sigma)$. Then the left-hand side of 12 will have the same value at $\widehat{y_0}s$ after the shock that it had at $\widehat{y_0}$ before the shock. However, $-\pi_2/\pi_1$ will be lower. The profit ratio is given by:

$$\frac{-\pi_2}{\pi_1} = \frac{w + c(q + r) - p_2 s}{p_1 s - w - c(q + r)} \tag{18}$$

An increase in s reduces the numerator and raises the denominator, reducing the ratio on the right side of 12. After the positive productivity shock, the new equilibrium value of the employment criterion will thus be less than $\widehat{y_0}s$. The effect of a productivity shock can be understood using Figure 1. If the horizontal axis is now y_0/s, the curve representing the left-hand side of 12 does not move in response to a productivity shock. Only the horizontal line at $-\pi_2/\pi_1$ is affected by s, and it moves down when s goes up. The new equilibrium market criterion will be less than $\widehat{y_0}s$. If y_s is the equilibrium employment criterion after the shock, then $y_s/s < y_0$ and $F(y_s/s - p_i; \sigma) < F(y_0 - p_i; \sigma)$. The effect on the unemployment rates is therefore the same

as a reduction in the employment criterion, holding the productivity shock fixed. The unemployment rates from both types of workers decline, and from Figure 3, when the unemployment rates are sufficiently low, the reduction in the employment criterion raises u_1/u_2. From 11, it follows that μ also increases. This completes the proof.

3.3 Average Productivity

The most important consequence of Theorem 3 is that the change in the mix of employed will have opposite effects from the productivity shock itself. Although productivity shocks will substantially affect the aggregate levels of employment and unemployment, the observed effect on average productivity will be substantially moderated. While a positive productivity shock will by itself raise average productivity, the increase in the proportion of employed who are low productivity will reduce it. Then substantial fluctuations in employment could be associated with negligible or undetectable productivity changes. This section examines the effects of productivity shocks on average productivity.

The proportion of employed workers who are high productivity is given by

$$\nu = \frac{\rho(1-u_1)}{\rho(1-u_1)+(1-\rho)(1-u_2)} \tag{19}$$

Then average productivity, ϕ, is

$$\phi = \nu p_1 s + (1-\nu) p_2 s \tag{20}$$

The average productivity depends both on the productivity shock, s, and on ν. In turn, ν depends on the ratio of employment rates:

$$\nu = \frac{\rho(1-u_1)/(1-u_2)}{\rho(1-u_1)/(1-u_2)+(1-\rho)} \tag{21}$$

The ratio of employment rates can be rewritten as

$$\frac{1-u_1}{1-u_2} = 1 + \frac{1-u_1/u_2}{(1/u_2)-1} \tag{22}$$

When $1/u_2$ is substantially greater than one, the ratio of employment rates decreases when the ratio of unemployment rates, u_1/u_2, rises. The relations

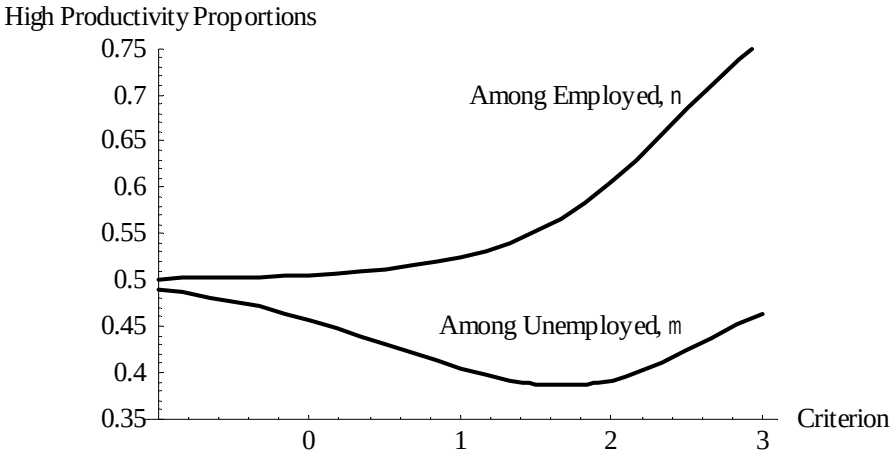

Figure 4: Proportions of Workers That Are High Productivity

between the employment criterion y_0 and μ and ν are shown in Figure 4, holding the productivity shock s fixed. As shown, a reduction in the employment criterion raises the proportion of unemployed that are high productivity and reduces the proportion of employed that are high productivity, if the unemployment rates are sufficiently low.

From Theorem 3, a positive productivity shock reduces y_0/s. From 17, the effect of a productivity shock that reduces y_0/s is equivalent to a reduction in y_0, holding s fixed. As a result, ν declines in response to a positive productivity shock. The net effect on ϕ in 20 is summarized in the following theorem:

Theorem 4 *In response to a productivity shock $s > 1$ (starting from $s = 1$), observed average productivity increases less than proportionately to the productivity shock when unemployment rates are sufficiently small.*

Figure 5 shows the relationship between average productivity and the productivity shock using the same assumptions as in Figure 1. As shown, starting at low levels of s, a positive productivity shock reduces average productivity instead of raising it.

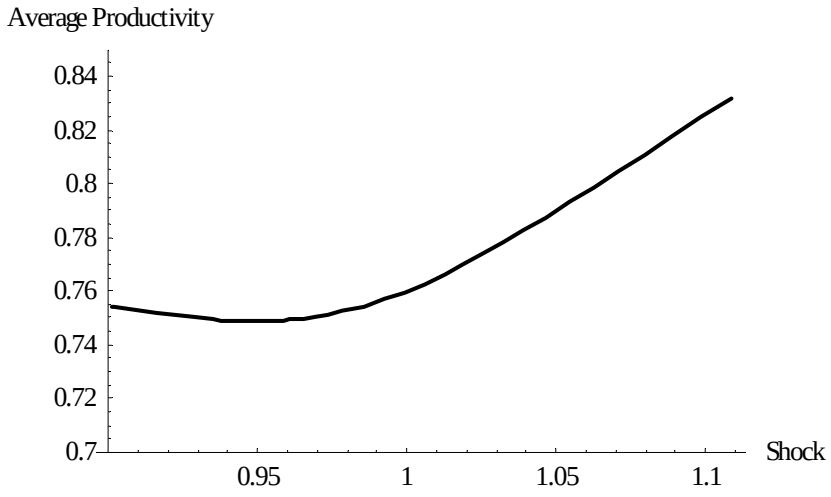

Figure 5: Response of Average Productivity to Shock

4 Comparisons of Labor Market Disturbances

This section demonstrates that the relation between employment and average productivity depends on the source of disturbances to the economy. The previous section developed the consequences of productivity shocks in the employment criterion model, showing that a positive productivity shock generates an increase in employment (or decrease in unemployment) in combination with an observed average productivity change that is smaller than the productivity shock. An alternative disturbance to the economy in the employment criterion model is a fluctuation in the interest or discount rate, presumably caused by a monetary disturbance. Fluctuations in the interest rate will be shown to generate a negative relation between average productivity and employment, just the opposite of the relation generated by productivity shocks.

Effects of interest rate fluctuations are simpler to analyze than productivity shocks. A change in the interest rate has no effects on the left-hand side of the market equilibrium condition in 12. From 18, an increase in the interest rate raises the ratio $-\pi_2/\pi_1$. In Figure 1, the upward sloping line stays fixed while the horizontal line at the level $-\pi_2/\pi_1$ goes up, so that the market equilibrium employment criterion is higher. The higher employment

criterion then raises the unemployment rates of both types of workers and, if the unemployment rates are sufficiently low, lowers the ratio u_1/u_2, lowers the proportion of high productivity workers among the unemployed, μ, and raises the proportion of high productivity workers among the employed, ν. With no change in the productivities of high and low productivity workers (i.e., there is no productivity shock), the average productivity ϕ goes up. The decline in employment is then associated with an increase in average productivity. These results are summarized in the following theorem.

Theorem 5 *In the Employment Criterion Model, a higher interest rate yields a higher employment criterion, higher unemployment rates and lower employment. If the unemployment rates are sufficiently small, a higher interest rate yields a higher proportion of high productivity workers among the employed, ν, and higher average productivity.*

The possibility of opposite relationships between employment and average productivity, depending on the source of the disturbance, explains weak or contradictory evidence of the cyclicality of productivity. The possibility of opposite relationships is summarized in the following theorem.

Theorem 6 *In the Employment Criterion Model (with the wage rate fixed), an increase in employment can occur with an increase or a decrease in average productivity depending on whether the source of the disturbance is a productivity shock or a fluctuation in the interest rate.*

Figure 6 shows the two different relations between average productivity and the aggregate employment rate, given by $\rho(1-u_1)+(1-\rho)(1-u_2)$. Consistent with Theorem 6, productivity shocks generate a negative and positive relation between employment and average productivity, while interest rate fluctuations generate a negative relation.

Another disturbance often discussed in the macroeconomic literature is a change in the real wage. With homogeneous labor, an exogenous increase in the real wage reduces employment by moving firms back up their derived demand curves. The lower employment levels result in higher average labor productivity (since the capital to labor ratio is increased). In the model developed here (with heterogeneous labor and an employment criterion), an increase in the wage has the same effects as an increase in the interest rate. The employment criterion must be higher to satisfy 12, employment decreases, and the average productivity increases. The consequences of a wage

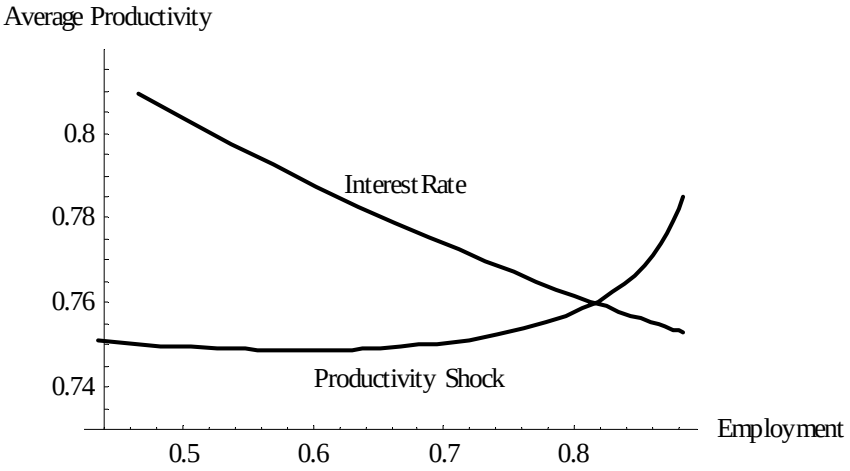

Figure 6: Alternative Relations Between Employment and Average Productivity

increase are therefore the same as in the standard macroeconomic analysis, even though the productivity of a given worker does not decline as more workers are hired. Wage rate fluctuations (holding the productivity shock and interest rate fixed) generate the same relationship between employment and average productivity as interest rate fluctuations.

In the analysis of Sections 2 and 3, the wage has been taken to be exogenously determined. If productivity shocks have only short term effects, they can be expected to have negligible effects on the wage rate and this assumption is reasonable. If on the other hand a productivity shock has a lasting effect, then eventually the wage would adjust. Wage adjustment to a continuing productivity shock can be determined from the aggregate condition that in the long run, the wage must be such that firm profits are zero. Using this approach, the firm profit expression in 8 can be set equal to zero and solved for the wage w as a function of the employment condition, y_0. The wage rate generated by the resulting function is such that firm profits are zero. This relation can then be combined with the relation generated by the market condition 12 to yield the long run determination of the wage. This is demonstrated in Figure 7. The downward sloping curve shows combinations of w and y_0 that yield zero firm profits. From 8, the firm faces a cost for

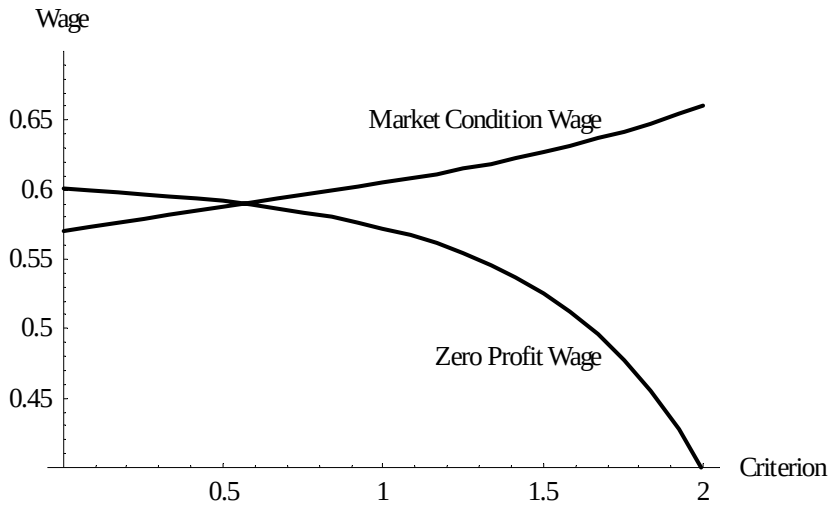

Figure 7: Determination of Wage Rate

every worker interviewed, C_I. If the firm hires fewer workers (because of a higher employment criterion), then the profit on each hired worker must be greater. This in turn requires that the wage be lower, generating the downward sloping zero profit curve in Figure 7. The upward sloping curve in the figure arises from the market condition 12. At higher wage rates, the profits from the high and low productivity workers are lower, leading firms to choose a higher employment criterion. The intersection of the two curves yields the wage rate and employment criterion consistent with long run equilibrium.

5 Conclusions

The phenomenon that drives the conclusions of this paper is that the mix of workers changes in response to productivity shocks. In response to a positive productivity shock, the proportion of employed workers that are low productivity increases. This change in the mix of workers has an effect on the average productivity that is opposite to the productivity shock itself. As a result, there can be a substantial and positive employment response to a positive productivity shock without a large observed increase in average productivity.

The change in the mix of workers arises because firms in the paradigmatic

model face a problem of choosing an employment criterion rather than an amount of labor to hire at a given wage rate. With the employment criterion as the variable subject to firm control, labor market reactions to productivity shocks take the form of adjustments in the employment criterion rather than direct changes in employment and, indirectly, in wage rates. The fluctuations in the employment criterion yield the changes in mix of workers and employment. Wage changes take place through a process that may take longer than the adjustments in firm employment criteria.

The weak relationship between employment and observed average productivity is demonstrated in the case worked out in the paper, in which the positive productivity shock results in a decline in the employment criterion (instead of just an increase that is smaller than the productivity shock, the outcome proven in Theorem 4). Then employment increases both because of the positive productivity shock and because of the reduction in the criterion.

Opposite relations between employment and observed average productivity can also arise because of different sources of disturbances in the model. Productivity shocks can yield a positive but weak relationship between employment and observed average productivity, while disturbances that generate a fluctuation in the interest rate could yield a negative relationship. Estimates of correlations would then be sensitive to time periods included.

The employment criterion model developed here provides a simple means to explain observed relationships among major macroeconomic variables, including employment, wage rates and productivity, that are inconsistent with a simple homogeneous worker view of the labor market. When firms use employment criteria as the margin of adjustment during business cycles, productivity shocks can generate large fluctuations in employment with no strong correlation between observed productivity and employment.

References

[1] Benhabib, Jess, Rogerson, Richard, Wright, Randall, 1991. Homework in Macroeconomics: Household Production and Aggregate Fluctuations, Journal of Political Economy 99, 1166-1187.

[2] Cho, Jang-Ok, 1995. Ex post Heterogeneity and the Business Cycle, Journal of Economic Dynamics and Control 19, 533-551.

[3] Cho, F. , Rogerson, Richard D., 1988. Family Labor Supply and Aggregate Fluctuations, Journal of Monetary Economics 21, 233-245.

[4] Christiano, Lawrence J., Eichenbaum, Martin, 1992. Current Real Business Cycle Theories and Aggregate Labor Market Fluctuations, American Economic Review 82, 430-450.

[5] Gaston, Robert, 1972. Labor Market Conditions and Employer Hiring Standards, Industrial Relations 11, 272-278.

[6] Hansen, Gary D., 1985. Indivisible Labor and the Business Cycle, Journal of Monetary Economics, Vol. 16, 309-327.

[7] Hansen, Gary D., Sargent, Thomas, 1988. Straight Time and Overtime in Equilibrium, Journal of Monetary Economics 21, 281-308.

[8] Hansen, Gary D., Wright, Randall, 1992. The Labor Market in Real Business Cycle Theory, Federal Reserve Bank of Minneapolis Quarterly Review 16, 2-12.

[9] Hartley, James E., Hoover, Kevin D., Salyer, Kevin D., eds, 1998. Real Business Cycles: A Reader, (Routledge, New York).

[10] King, Robert G., Plosser, Charles, Rebelo, Sergio, 1988. Production, Growth and Business Cycles II. New Directions, Journal of Monetary Economics 21, 309-341.

[11] Kydland, Finn E., 1984. Labor Force Heterogeneity and the Business Cycle, Carnegie-Rochester Conference Series on Public Policy 21, 173-208.

[12] Kydland, Finn E., Prescott, Edward C., 1982. Time to Build and Aggregate Fluctuations, Econometrica 50, 1345–1369.

[13] Lockwood, Ben, 1991. Information Externalities in the Labor Market and the Duration of Unemployment, Review of Economic Studies 58, 733-53.

[14] Plosser Charles I., 1989. Understanding Real Business Cycles, Journal of Economic Perspectives 3, 51-77.

[15] Prasad, Eswar, 1996. Skill Heterogeneity and the Business Cycle, Canadian Journal of Economics XXIX, 910-929.

[16] Schlicht, Ekkehart, 1981. Training Costs and Wage Differentials in the Theory of Job Competition, Zeitschrift fur die Gesamte Staatswissenschaft 137, 212-221.

[17] ———————, 2002. Hiring Standards and Market Clearing, Institute for the Study of Labor (IZA) Discussion Paper No. 481, Bonn.

[18] Thurow, Lester, 1975. Generating Inequality: Mechanisms of Distribution in the US Economy, (Basic Books: New York).